EFFECT OF CORRELATIONS ON COHERENT AND INCOHERENT PROCESSES IN NUCLEI ‡

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The results of calculating the effect of correlations in the nuclear wave function on elastic and inelastic scattering of high energy hadrons are described. One can also make use of these results to calculate coherent and incoherent photo-production processes.

We have calculated the effects of including two body correlations in the nuclear wave function on elastic and inelastic scattering of hadrons and on coherent and incoherent photo-production. We allow for the form factors of the corresponding two body scattering and production processes writing these as $f(t) = f(0)\,e^{\frac{1}{2}at}$ where $t = -q^2$ is the square of the four momentum transfer. We first review results neglecting the correlations and the t dependence of $f(t)$. We neglect spin and iso-spin dependence, taking the ground state wave function squared as

$$|u_{\mathrm{I}}(\boldsymbol{r}_1,\dots,\boldsymbol{r}_A)|^2 = \prod_{k=1}^{A} \rho(\boldsymbol{r}_k) \tag{1}$$

For medium to heavy nuclei this leads to an elastic scattering amplitude [1], or coherent production amplitude [2] at energies where longitudinal momentum transfer is unimportant,

$$F^{(\mathrm{c})}(q^2) = f(0)\,N(q\,;0,\tfrac{1}{2}\sigma') \tag{2}$$

$$N(q\,;0,\tfrac{1}{2}\sigma') = \frac{2}{\sigma'}\int \exp(\mathrm{i}q\cdot b)\,\mathrm{d}^2b[1-\exp\{-\tfrac{1}{2}\sigma' T(b)\}] \tag{3}$$

where $T(b) = A\int_{-\infty}^{\infty}(\boldsymbol{b},z)\,\mathrm{d}z$, $\sigma' = \sigma[1-\mathrm{i}\,\mathrm{Re}f(0)/\mathrm{Im}\,f(0)]$. Using closure and assuming that only one inelastic step is important, the differential cross section including excitation of all nuclear states is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(0)|^2\{|N(q\,;0,\tfrac{1}{2}\sigma')|^2 + N_1(0\,;\sigma) + \frac{2}{A}\mathrm{Re}[N_2(q\,;\tfrac{1}{2}\sigma')N^*(q\,;0,\tfrac{1}{2}\sigma')] + -\frac{1}{A}|N_1(q\,;\tfrac{1}{2}\sigma')|^2\} \tag{4}$$

$$N_m(q\,;\sigma) = \frac{1}{m!}\frac{1}{\sigma}\int \exp(\mathrm{i}q\cdot b)[\sigma T(b)]^m \exp(-\sigma T(b)\mathrm{d}^2b \tag{5}$$

In first Born approximation this becomes (let $\sigma \to 0$)

$$\sigma_{\mathrm{B}}/\mathrm{d}\Omega = |f(0)|^2\{A^2|F(q)|^2 + A[1-|F(q)|^2]\} \tag{6}$$

where $F(q) = \int \exp(\mathrm{i}q\cdot r)\,q(\boldsymbol{r})\,\mathrm{d}^3r$.

We now include nuclear wave function correlations and the form factor of the two body amplitude $f(t)$. It is important that we consider these together since the range of the two body interaction producing $f(t)$ is not much smaller than the range of the correlations. We define a two body correlation function $g(r_i, r_j)$ through the equations

$$\int|u_{\mathrm{I}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3,\dots\boldsymbol{r}_A)|^2\,\mathrm{d}\boldsymbol{r}_3\,\mathrm{d}\boldsymbol{r}_4\dots\mathrm{d}\boldsymbol{r}_A \equiv \equiv \rho(\boldsymbol{r}_1)\rho(\boldsymbol{r}_2)[1+g(\boldsymbol{r}_1,\boldsymbol{r}_2)] \tag{7}$$

$$\rho(\boldsymbol{r}_1) = \int|u_{\mathrm{I}}|\,\boldsymbol{r}_1,\boldsymbol{r}_2,\dots\boldsymbol{r}_A)|^2\,\mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3\dots\mathrm{d}\boldsymbol{r}_A \tag{7a}$$

We find then, neglecting higher order correlations, that eq. (4) is to good approximation replaced by

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$$\frac{d\sigma}{d\Omega} = |f(0)|^2 \{ |M(q\,;0,\tfrac{1}{2}\sigma')|^2 + \\ + \frac{2}{A}\mathrm{Re}[M_2(q,\tfrac{1}{2}\sigma')M^*(q\,;0,\tfrac{1}{2}\sigma')] + \\ - \frac{1}{A}|M_1(q,\tfrac{1}{2}\sigma')|^2\} + \frac{d\sigma^{(I)}}{d\Omega} \tag{8}$$

$$\frac{d\sigma^{(I)}}{d\Omega} \equiv |f(t)|^2 N_{\mathrm{eff}}(\sigma,\xi)[1+\eta(\sigma)G(t)] \tag{9}$$

$$N_{\mathrm{eff}}(\sigma,\xi) = \frac{1}{\sigma}\int d^2b\,[\sigma T(b) - 4\xi\sigma^2 Q(b)]\exp(-\sigma T_R(b)) \tag{10}$$

$$G(t) = \int g(\boldsymbol{r})\exp(\mathrm{i}q\cdot r)\,d^3r \tag{11}$$

$$\eta(\sigma) = \frac{\int \exp(-\sigma T(b)\,Q(b)\,d^2b}{\int \exp(-\sigma T(b)\,T(b)\,d^2b}\,,$$
$$Q(b) = A^2\int_{-\infty}^{\infty}\rho^2(b,z)\,dz \tag{12}$$

We have assumed here that $g(\boldsymbol{r}_1, \boldsymbol{r}_2) \approx \approx g(\boldsymbol{r}_1 - \boldsymbol{r}_2)$. Table 1 lists values of η. The quantities M, M_1 and M_2 in eq. (8) are obtained from N, N_1 and N_2 respectively of eqs. (3) and (5) through the replacement of $T(b)$ by

$$T_R(b) = T(b) - \xi\,Q(b)\,\sigma \tag{13}$$

$$\xi = \frac{1}{16\pi a}\int \exp(-b^2/4a)\,g(\boldsymbol{b},z)\,d^2b\,dz \tag{14}$$

The correlation length ξ can in principle be obtained from nuclear models which yield $g(\boldsymbol{r})$. Again the Fourier transform of $g(\boldsymbol{r})$ appears prominently in the incoherent term

$$\frac{1}{|f(t)|^2}\frac{1}{N_{\mathrm{eff}}(\sigma,\xi)}\frac{d\sigma^{(I)}}{dt} = 1 + \eta(\sigma)\,G(t) \tag{15}$$

A plot of the t dependence of the left hand side of eq. (15) would yield $G(t)$ and hence its Fourier transform and therefore ξ. Unfortunately the coherent terms dominate in eq. (8) at small t and make the determination of $G(t)$ from scattering experiments difficult if not impossible. However, photo-production of positive pions is governed by an expression similar to (15). Since it is a charge exchange process, coherent terms vanish and so correlation function information can be obtained when data are available. We have made estimates of ξ from nuclear models and find it to be $\sim$ -0.3 to -0.4 fm using a value for $a = 8(\mathrm{GeV}/c)^{-2}$. It is to be noted that the t dependence of $f(t)$ diminishes the effect of correlations. We find the decrease to be roughly a factor of two. The negative sign for ξ results from the repulsive core at small inter-nucleon distances and the Pauli principle. Our estimates come from taking $g(0) = -1$ and using simple gaussian forms of range about 1 fm, including some weak attraction.

We now have sufficient information to compute the effect of short range correlations on the diffractive parts of eq. (8) which fall rapidly with momentum transfer. The detailed prescription is given in formula (13). In addition, the two body form factor acts to produce small changes in the effective nuclear radial shape parameters. This can be looked after by adjusting these parameters to fit the diffractive slope for each nucleus. This slope does not change much for small changes in σ.

We can then write

$$M(0;0,\tfrac{1}{2}\sigma) = \\ = N(0\,;0,\tfrac{1}{2}\sigma)\,\xi\sigma\int\exp(-\tfrac{1}{2}\sigma T(b)\,Q(b)\,d^2b \equiv \\ \equiv N(0\,;0,\tfrac{1}{2}\sigma_R)\,. \tag{16}$$

This renormalized σ_R is then slightly smaller (larger) than σ for ξ negative (positive). Alternatively, we can write the coherent production amplitude in terms of an effective two body amplitude $f^{(E)}(0) \equiv f(0)\,\sigma_E/\sigma$ in the form

$$F^{(c)}(t) = f(0)\,M(q\,;0,\tfrac{1}{2}\sigma) \equiv f^{(E)}(0)\,N\ q\,;0,\tfrac{1}{2}\sigma_E) \tag{17}$$

which serves to define σ_E which is slightly larger (smaller) than σ for ξ negative (positive). Other terms in eq. (8) can be expressed in terms of σ_E in a similar way. Using the right hand expression for $F^{(c)}(t)$ in eq. (17) is equivalent to using an optical model with potentials $U_{\alpha\beta}$ =

Table 1

The quantity $10\eta(\sigma)$ in $(\mathrm{fm})^{-3}$ for a nuclear density $\rho(r) = \rho_0/[1+\exp\{(r-b)/c\}]$, $b = 1.14A^{\frac{1}{3}}$ fm and $d = 0.545$ fm.

A \ σ(mb)	5	7.5	10	12.5	15	20	25	30	35	40	50
208	1.07	1.02	0.964	0.900	0.837	0.713	0.600	0.505	0.428	0.369	0.285
108	0.979	0.938	0.894	0.848	0.800	0.705	0.613	0.531	0.460	0.401	0.314
64	0.889	0.856	0.823	0.786	0.748	0.675	0.598	0.535	0.474	0.420	0.336
27	0.717	0.697	0.675	0.654	0.631	0.587	0.542	0.499	0.458	0.420	0.352
20	0.654	0.636	0.620	0.600	0.582	0.546	0.510	0.474	0.440	0.407	0.349

$= -4\pi f^{(E)}_{\alpha\beta} A\,\rho(\boldsymbol{b}, z)$ and this allows for the calculation of production processes at finite energy. The coherent amplitude for photo-production of diffractively produced mesons can then be written [2]

$$F^{(c)}(t) = f^{(E)}(0)\,A \int J_0(q_t b)\exp\{\mathrm{i}q_l z\}\,\rho(b,z)\,\times \times \exp\{-\tfrac{1}{2}\sigma'_E A \int_z^\infty \rho(b,z')\,\mathrm{d}z'\}\,\mathrm{d}^2 b\,\mathrm{d}z\,. \tag{18}$$

We find, using the eqs. (3), (13) and (17) that a good approximation to σ_E is given by the A dependent expression

$$\sigma_E = \sigma[1 - \xi\eta(\tfrac{1}{2}\sigma)\,\sigma] \tag{19}$$

It is of interest to see what the difference is between σ_E and σ. For $A = 208$, using the value $\xi = -0.4$ fm, we get for $\sigma = 25$ mb, $\sigma_E = 27.3$ mb; for $\sigma = 30$ mb, $\sigma_E = 33.0$ mb and for $\sigma = 40$ mb, $\sigma_E = 42.4$. Differences of this size can perhaps be detected with very careful experiments on nuclei using particles whose scattering amplitudes on nucleons are well determined.

Finally we point out that our expressions for coherent and incoherent correlation effects differ in detail from those of other authors which have been produced recently [3]. In general our corrections are smaller.

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